Pole and Continuum Contributions to Proton-Proton Scattering*

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The $N\bar{N}\to 2\pi$ s and p wave and ω contributions to proton-proton scattering have been recomputed, using the scheme of Amati, Leader, and Vitale. The previous anomalies in the phase shifts have disappeared. In addition, the 2π s-wave contribution of Bowcock, Cottingham, and Lurié is shown to be indistinguishable in effect from the exchange of a scalar meson with a mass of about 400 MeV. However, the 2π d and higher wave contributions of Chew, Goldberger, Low, and Nambu cannot be accurately represented by the exchange of scalar mesons. Either (i) the higher wave contributions calculated by Amati, Leader, and Vitale contain numerical errors, or (ii) a different mechanism is needed for the 2π s- and p-wave contributions, or (iii) the theoretical approximations used are not sufficient.

I. INTRODUCTION

THERE have been several attempts within the past few years to use multipion resonances for reproducing various features of proton-proton scattering. In most of the models, resonant two- and three-pion states were treated as mesons of well-defined mass. Bryan, Dismukes, and Ramsay[1] (BDR) used the exchange of single π, vector, and scalar mesons in order to calculate a *potential*. They found that the vector meson produced the desired repulsion in the interior region of the potential; they introduced the scalar meson to provide attraction in the intermediate range of 1–2 F. Sawada, Ueda, Watari, and Yonezawa[2] (SUWY) used the same three mesons, calculating pp phase shifts from the Born approximation with a rescattering correction which made their amplitudes unitary. They too were led by phenomenology to include a scalar meson. Scotti and Wong[3] added the ω and η for a total of five mesons, and unitarized their amplitudes by the use of a once-subtracted dispersion relation. The amplitudes were also given a Regge-type behavior at high energy in order to render the dispersion integrals convergent.

Naively, one might expect that uncorrelated multipion exchanges could also have appreciable contributions to the two-nucleon interaction. Amati, Leader, and Vitale[4] (ALV) have used the Cini-Fubini approximation to the Mandelstam representation in order to calculate the *total* contributions from the exchange of two pions. ALV assumed that the Chew, Goldberger, Low, and Nambu (CGLN) $N\bar{N}\to 2\pi$ amplitude, obtained from the CGLN[5] pion-nucleon amplitude, would be almost exact for angular momenta $L\geq 2$. Hence, the d and higher wave contributions from the CGLN amplitude were labeled "2π basic" by ALV, indicating their supposed independence of any explicit $\pi\pi$ model. The $N\bar{N}\to 2\pi$ s and p waves, however, could by strongly dependent on the assumed model for $\pi\pi$ scattering. The BCL amplitude was used by ALV for the s-wave $2\pi(N\bar{N}\to 2\pi)$ amplitude, and finally a single ρ exchange was used for the p wave: they had found that use of the CGLN p-wave amplitude resulted in too much attraction in the pp system. ALV also included the effect of a strong three-pion resonance by adding in a one ω-exchange amplitude. A convenient summary of the theory and notation is provided in the latest ALV communication.[4]

It has been noted[6] that there appear to be discrepancies in the results published by ALV. In particular, the published $^3F_2$ and ϵ_4 phases do not become one-pion exchange at low energy, whereas centrifugal barrier arguments lead one to expect the contrary. In addition, ALV's ω contribution seems to have an energy-dependent coupling constant which is everywhere quite far from the quoted value.

With the above noted discrepancies in mind, the 2π s- and p-wave contributions have been recalculated. After adding in the higher wave "2π basic" contributions calculated by ALV, the discrepancies were found to have disappeared. At the same time, a degree of reconciliation with the "poles only" models has been achieved by demonstrating the effective equivalence of the scalar meson mechanism to that of the BCL 2π s wave.

II. CALCULATION AND RESULTS

The ALV 2π s-wave contribution comes from the BCL pion-nucleon amplitudes[4]

$$A^+(s,t)=\frac{1}{\pi}\int_{(m+\mu)^2}^{\infty}\sigma_A{}^+(s',t)\left[\frac{1}{s'-s}+\frac{1}{s'-\bar{s}}\right]ds'+C_A{}^+,$$

$$B^+(s,t)=\frac{1}{\pi}\int_{(m)^2}^{\infty}\bar{\sigma}_B{}^+(s',t)\left[\frac{1}{s'-s}-\frac{1}{s'-\bar{s}}\right]ds'.$$

We have calculated the contributions of these amplitudes to the various pp phase shifts for $L\geq 2$. The latter

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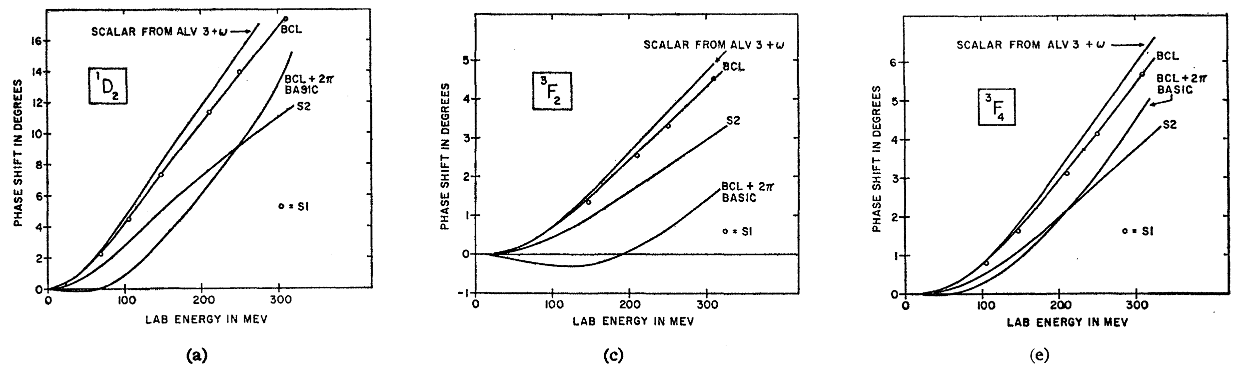
[2] S. Sawada, T. Ueda, W. Watari, and M. Yonezawa, Progr. Theoret. Phys. (Kyoto) **28**, 991 (1962), hereafter SUWY.

[3] A. Scotti and D. Y. Wong, Phys. Rev. Letters **10**, 142 (1963).

[4] D. Amati, E. Leader, and B. Vitale, Nuovo Cimento **17**, 68 (1960); **18**, 409 (1960); Phys. Rev. **130**, 750 (1963).

[5] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1337 (1957).

[6] P. Signell and D. G. Marker, Phys. Rev. **134**, B365 (1964); P. Signell, Phys. Rev. **133**, B982 (1964).



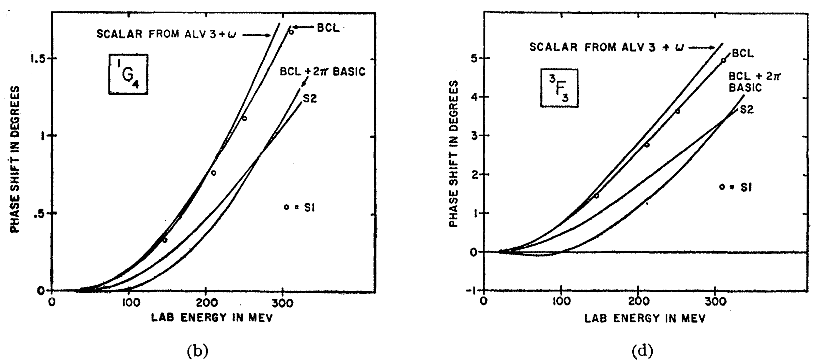


FIG. 1. Proton-proton scattering phase shifts produced by various 2π s-wave contributions alone. The masses and coupling constants of the scalar mesons S1 and S2 are shown in Table I.

are labeled "BCL" in Fig. 1. The circles shown in the figure are the phases resulting from the exchange of one scalar meson of mass 400 MeV and coupling constant $g_s^2/4\pi=4.724$. It is seen that *the two models produce virtually identical phases in all states*. That is, the effect on pp scattering of the BCL mechanism is indistinguishable from the effect of a scalar meson exchange.

One is naturally led to inquire as to whether *all* of the non-ρ 2π contributions are representable as a scalar meson exchange. If so, complete correspondence would be achieved between the phenomenological models and the theoretical one. Unfortunately, such is not the case. As is seen in Fig. 1, the "BCL$+2\pi$ basic" contributions have an energy dependence which is quite different from that which is produced by single scalar meson exchanges. In order to compare shapes, Fig. 1 shows the prediction for a 400-MeV scalar meson with coupling constant adjusted to yield phases of the same order of magnitude as "BCL$+2\pi$ basic." Note that neither a change of the scalar mass nor the addition of another scalar meson could reproduce the "BCL$+2\pi$ basic" curves. Also shown in Fig. 1 are the phases produced by one scalar exchange where the coupling constant was adjusted in data fitting to be described below.

Figure 2 displays the ALV *published* phases, here labeled "ALV $1+\omega$." The ALV curve for the $^1D_2$ phase without an ω contribution was deduced from ALV's Fig. 1(a). The experimental points shown in Fig. 2 are those deduced via "modified phase-shift analyses[6]" of the pp data. The phases, predicted by combining the numbers in the ALV "2π basic" tables with our recalculated 2π s- and p-wave contributions, are labeled "ALV 2" in the figures. The addition of an ω meson, "ALV $2+\omega$," with its coupling constant adjusted so as to give a least-squares fit to the "data," yields an improved fit.

Comparison of the "ALV $1+\omega$" and "ALV $2+\omega$" curves in Figs. 2(c) and 2(f) shows immediately that the strange energy dependence displayed by the original ("ALV $1+\omega$") $^3F_2$ and ϵ_4 phases must have been due to a calculational error. The effect of the centrifugal barrier is now apparent. It is interesting to note that the original unusual (but incorrect) $^3F_2$ energy dependence was thought at the time to be of positive merit.[4]

The three phenomenological models described in the introduction used pole terms only. If one takes these as serious models for the contribution of the $\pi\pi$ interaction, then one should discard the BCL amplitude and take the 2π s and p waves as given by scalar and ρ exchanges. One still has in addition the "2π basic" contribution from $N\bar{N}\rightarrow 2\pi$ waves of $L\geq 2$. Combining all of these, and adjusting the scalar and ω coupling constants for a least-squares fit to the "data," one obtains the curves labeled "ALV $3+\omega$" in Fig. 2. The contributions from the scalar alone are shown in Fig. 1.

III. CONCLUSIONS

With all contributions recalculated, except for the "2π basic" part, the pp phase shifts for $L\geq 2$ became one-pion exchange at low energy.

It was found that the effect of a BCL s-wave contribution is indistinguishable from that of one scalar meson

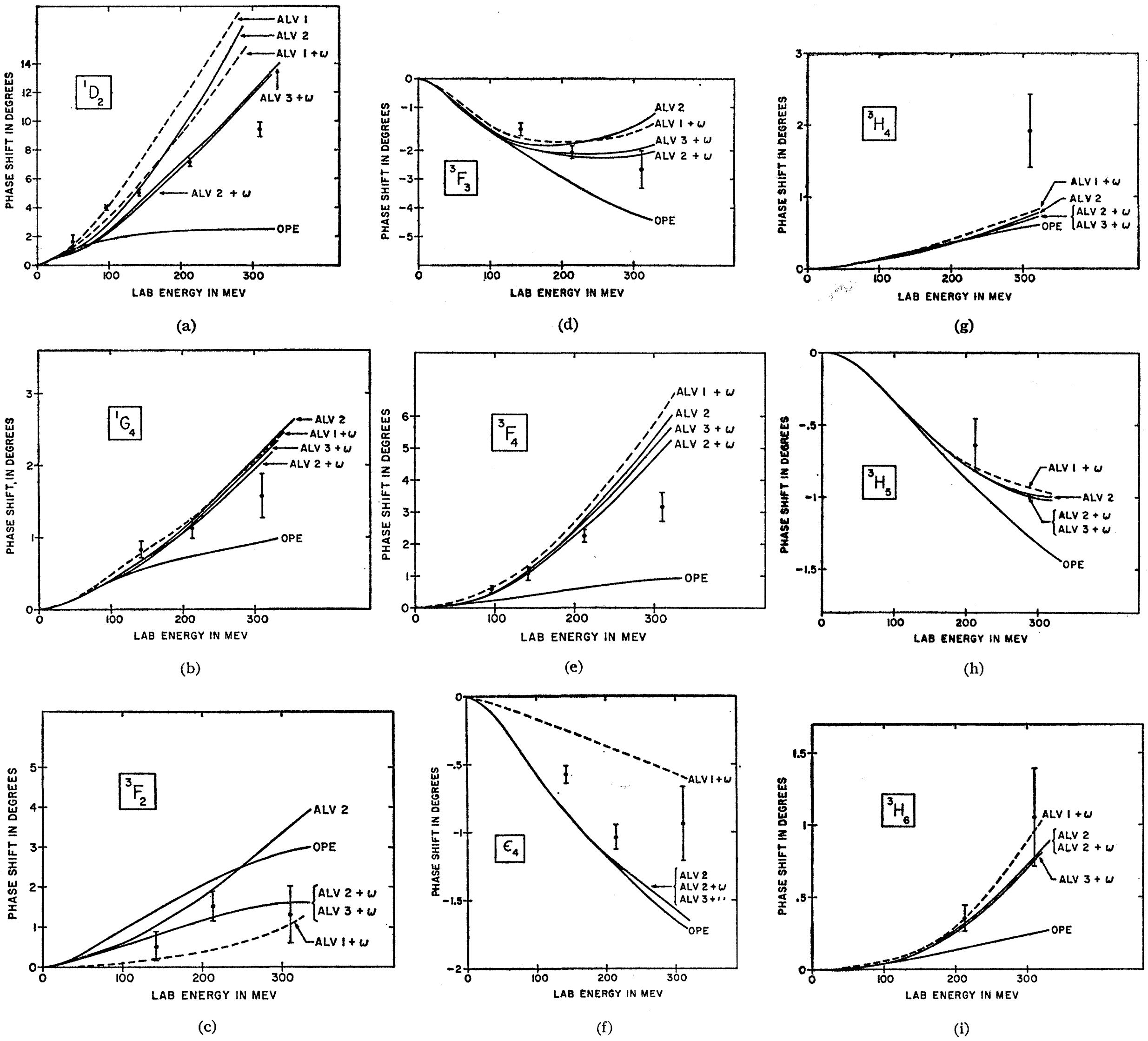


FIG. 2. The total phase shifts predicted by various models. The "experimental" points shown are from "modified phase-shift analyses" of the pp data.

exchange. There is some indirect evidence for the existence of a scalar "σ" meson with a mass of about 400 MeV, principally from η decay and the nucleon axial-vector form factor. Woo[7] has cited the use of a scalar meson in the phenomenological two-nucleon models as additional indirect evidence for the existence of the σ. The equivalent effect of the BCL s wave weakens that evidence.

If the Cini-Fubini approximation is valid and if the "2π basic" contributions have been correctly calculated by ALV, then two things are apparent. First, the "poles only" approach is insufficient since the d and higher waves of $N\bar{N} \rightarrow 2\pi$ make a large contribution to the nucleon-nucleon phase shifts, opposite in effect to the $N\bar{N} \rightarrow 2\pi$ s-wave contribution. Retaining the "2π basic" contribution forces one to have a substantially larger coupling constant for the scalar meson than that obtained if the "2π basic" were omitted. This is evident in Table I where, for instance, the scalar coupling constant is considerably larger for "ALV $3+\omega$" than for SUWY or Scotti-Wong. We have no explanation for the anomalously large coupling constants found by Bryan, Dismukes, and Ramsay.

The second consequence of the above assumptions is that the $N\bar{N} \rightarrow 2\pi$ s- and p-wave contributions cannot be accurately represented by the exchange of a scalar and a ρ meson. The discrepancy between the "ALV 3 $+\omega$" prediction and the data at, for instance, 96.5 and 142 MeV in Fig. 2(a) is not negligible. A better fit to the

[7] C. H. Woo, Phys. Rev. Letters **12**, 318 (1964).

TABLE I. Summary of masses and coupling constants for the mesons used in various proton-proton models. All the models include the one-pion contribution with $g_{\pi N}^2/4\pi=14.4$ and $m_\pi=135.1$ MeV, except BDR and Scotti-Wong who use $m_\pi=140$ MeV. For abbreviations, see text.

Model	Meson	$g_s^2/4\pi$	$g_v^2/4\pi$	$g_T^2/4\pi$	Mass (MeV)
BDR	Vector		30.	0.	760
	Scalar	13.8			560
SUWY	Vector		1.2	13.2	540
	Scalar	2.4			405
Scotti-Wong	ρ		1.27	11.39	591
	ω		2.77	0.	780
	Scalar	1.525			437
	η	12.12			550
	φ		2.26	0.	1020
ALV 2	ρ		0.84	11.6	750
ALV 2+ω	ρ		0.84	11.6	750
	ω		13.6	0.	780
ALV 3+ω	ρ		0.84	11.6	750
	ω		16.9	0.	780
	Scalar	5.16			400
S1	Scalar	4.72			400
S2	Scalar	3.22			400

data can be obtained from π, scalar, ρ and ω exchange only if the "2π basic" is neglected. If this is not done, then the only alternative to adding more mesons (adjustable parameters) is a more accurate treatment of the $N\bar{N}\rightarrow 2\pi$ s and p waves.

The calculations reported here were carried out at the Computation Center of The Pennsylvania State University.

APPENDIX

The coupling constants listed in Table I were defined in terms of the interactions[2]:

$$\mathrm{Pi}=g_\pi\bar{\psi}i\gamma_5\psi\varphi\,,$$

$$\mathrm{Scalar}=g_s\bar{\psi}\psi\varphi\,,$$

$$\mathrm{Vector}=g_v\bar{\psi}i\gamma_\mu\psi\varphi_\mu+(g_T/4m)\bar{\psi}\sigma_{\mu\nu}\psi\varphi_{\mu\nu}\,,$$

where:

$$\sigma_{\mu\nu}=1/2i(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)\,,$$

$$\varphi_{\mu\nu}=\partial_\mu\varphi_\nu-\partial_\nu\varphi_\mu\,,$$

$$m=\text{mass of proton}.$$

Then $g_\pi^2/4\pi=14.4$.

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Prediction of a $\pi\eta$ Resonance

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Using the bootstrap approximation of Zachariasen and Zemach, we predict a relatively broad 1^{--} resonance at about 1 BeV in a two-channel ($\pi\eta,\pi\rho$) calculation. Such a resonance can only belong to the isocuplet representation of SU(3). Tentatively an identification can be made on this basis with the B meson, and a recently observed peaking in the $\pi\rho$ channel at about 1250 MeV.

1. INTRODUCTION

THE pseudoscalar mesons—$\pi K\bar{K}\eta$—are now considered to belong to the octet representation of SU(3). A scattering state of two pseudoscalar mesons can be any of the 1, 8, 8′, 10, $\overline{10}$, or 27 dimension representations. For p-wave scattering, Bose statistics allow only the 8′, 10, and $\overline{10}$ states. It is observed that the well-known p-wave resonances—$\rho K^*\bar{K}^*\omega_8$—can be classified according to the 8′ representation. Naturally it is of interest to investigate whether the 10 and $\overline{10}$ p-wave states do not also resonate. In this connection Neville[1] has noticed that within the framework of a single-channel bootstrap calculation where all pseudoscalar and vector-meson masses are taken, respectively, equal, the tenfold resonant states should exist at the same mass as the observed octet state. We investigate this situation in somewhat more detail by looking at $\pi\eta$ scattering. The $\pi\eta$ state with odd parity cannot belong to 1, 8, 8′, or 27, but as we shall see must belong to the icosuplet state which consists of the 10 and its antiparticle state $\overline{10}$. Since this state is part of the multiplet, any general conclusion drawn about it should be valid for the entire multiplet. However, since in this note we do not look at all the decuplet channels, the calculation is to a large extent independent of SU(3).

We use the bootstrap technique to study the $l=1$, $J=1$, $I=1$, $G=-1$ amplitude. This method has the following history: Recently, Zachariasen[2] noticed that the appearance of the ρ resonance in $\pi\pi$ scattering could be qualitatively explained if it were assumed that the dominant force in this scattering comes from the exchange of a ρ. By taking the additional channel $\pi\pi\rightarrow\pi\omega$

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